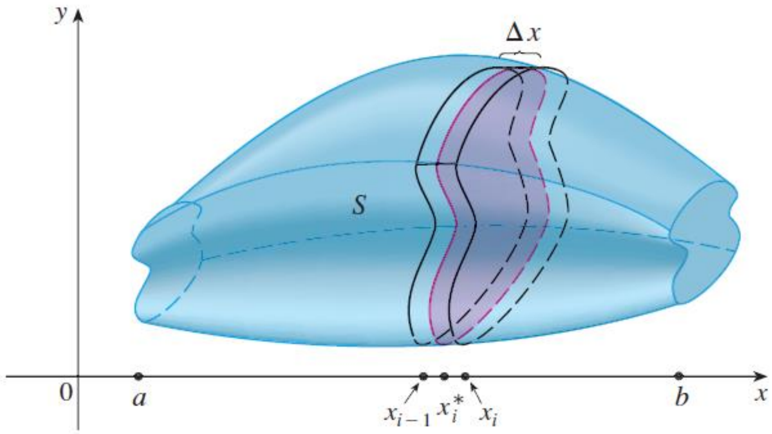
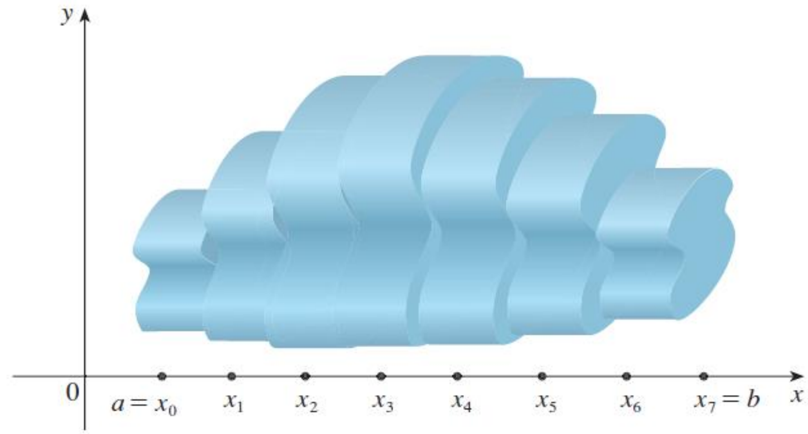
Closing today: HW_2A, 2B, 2C
Closing next Wed: HW_3A, 3B, 3C
(Pretend 3A, 3B, and 3C are closing Sunday!)
Midterm 1 is Thursday, April 21, covers 4.9, 5.1-5.5, 6.1-6.3

Entry Task:
Find the area of the region bounded by $x = y^2$ and $y = x^3$ in 2 ways:

(i) Using dx
(ii) Using dy

6.2 Finding Volumes Using Cross-Sectional Slicing





If we can find the general formula, $A(x_i)$, for the area of a cross-sectional slice, then we can approximate volume by:

$$\text{Volume of one slice} \approx A(x_i)\,\Delta x$$

$$\text{Total Volume} \approx \sum_{i=1}^{n} A(x_i)\,\Delta x$$

This approximation gets better and better with more subdivisions, so we say

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\,\Delta x$$

And we conclude

$$\text{Volume} = \int_a^b A(x)\,dx$$

$$= \int_a^b \text{"Cross-sectional area formula"}\,dx$$

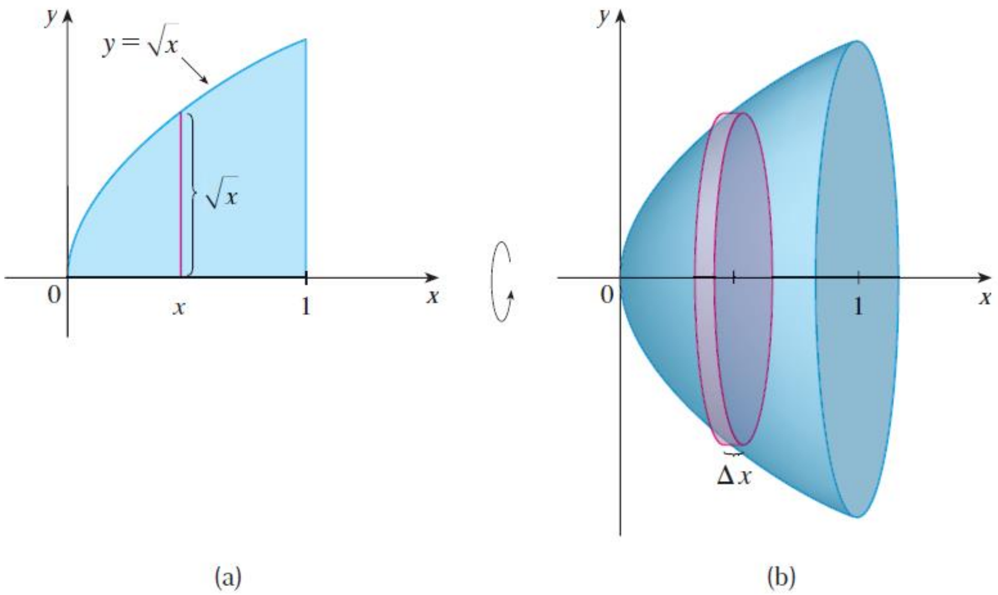
Volume using cross-sectional slicing

1. Draw! Cut **perpendicular** to the axis of rotation. If you draw a line at the cut, the axis you cut across is the variable you are using!
   Draw a typical cross-section, label Δx or Δy and label *x* or *y*, appropriately.
   Label everything in terms of the appropriate variable.
2. Area? Find the formula for the area of a cross-sectional slice.

Disc: Area = $\pi(\text{radius})^2$
Washer: Area = $\pi(\text{outer})^2 - \pi(\text{inner})^2$
Square: Area = (Height)(Length)
Triangle: Area = ½ (Height)(Length)

3. Integrate the area formula.

Example: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the x-axis.

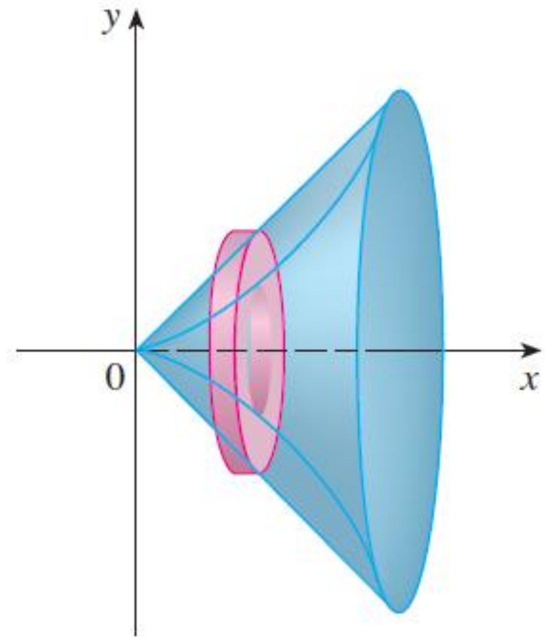


1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

Example: Consider the region, R, bounded by $y = \sqrt{x}$, x = 0, and y = 1. Find the volume of the solid obtained by rotating R about the y-axis.

Example: Consider the region, R, bounded by $y = x$ and $y = x^4$.
Find the volume of the solid obtained by rotating R about the x-axis.

1. Draw and label!
2. Cross-sectional area?
3. Integrate area.



Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. (R is the same as the last example).

(a) Now rotate about the horizontal line y = -5. What changes?

(b) Now rotate about the horizontal line y = 10. What changes?

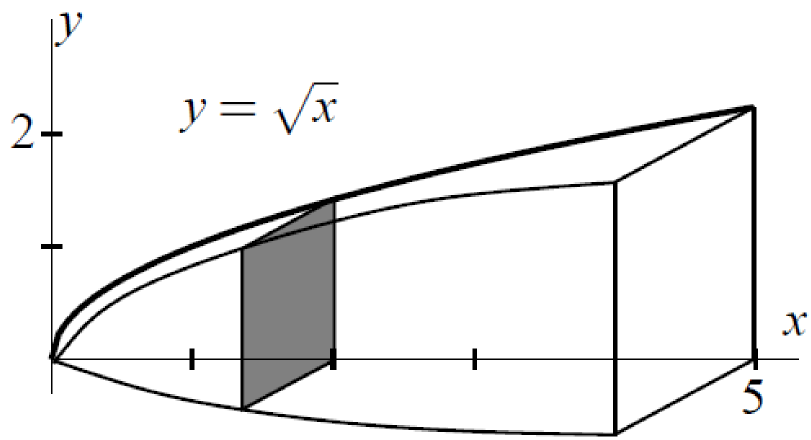
Example:
Set up an integral for find the volume obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the vertical line $x = -10$.

Example:

(From an old final and homework)

Find the volume of the solid shown.

The cross-sections are squares.



1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has one major limitation:
If the cross-sections are perpendicular to the x-axis (for example if you are rotating about the x-axis),
then you must use dx.

If the cross-sections are perpendicular to the y-axis (for example if you are rotating about the y-axis),
then you must use dy.

What if we were rotating about the x-axis and we wanted to use dy? This method won't work! We need another method. That is what we will do in 6.3.